$$\prod_{i=0}^{i<n}[x\cos(\frac{i\pi}{n})+y\sin(\frac{i\pi}{n})]-z=0$$

Für n = 10 gilt

$$[x\cos(\frac{0\pi}{10})+y\sin(\frac{0\pi}{10})][x\cos(\frac{1\pi}{10})+y\sin(\frac{1\pi}{10})]$$
$$[x\cos(\frac{2\pi}{10})+y\sin(\frac{2\pi}{10})][x\cos(\frac{3\pi}{10})+y\sin(\frac{3\pi}{10})]$$
$$[x\cos(\frac{4\pi}{10})+y\sin(\frac{4\pi}{10})][x\cos(\frac{5\pi}{10})+y\sin(\frac{5\pi}{10})]$$
$$[x\cos(\frac{6\pi}{10})+y\sin(\frac{6\pi}{10})][x\cos(\frac{7\pi}{10})+y\sin(\frac{7\pi}{10})]$$
$$[x\cos(\frac{8\pi}{10})+y\sin(\frac{8\pi}{10})][x\cos(\frac{9\pi}{10})+y\sin(\frac{9\pi}{10})]-z=0$$

Mit

$$\cos(\frac{0\pi}{10})=\cos(0°)=1$$
$$\cos(\frac{1\pi}{10})=\cos(18°)=\frac{\sqrt{10+2\sqrt{5}}}{4}$$
$$\cos(\frac{2\pi}{10})=\cos(36°)=\frac{\sqrt{5}+1}{4}$$
$$\cos(\frac{3\pi}{10})=\cos(54°)=\frac{\sqrt{10-2\sqrt{5}}}{4}$$
$$\cos(\frac{4\pi}{10})=\cos(72°)=\frac{\sqrt{5}-1}{4}$$
$$\cos(\frac{5\pi}{10})=\cos(90°)=0$$
$$\cos(\frac{6\pi}{10})=\cos(108°)=-\frac{\sqrt{5}-1}{4}$$
$$\cos(\frac{7\pi}{10})=\cos(126°)=-\frac{\sqrt{10-2\sqrt{5}}}{4}$$
$$\cos(\frac{8\pi}{10})=\cos(144°)=-\frac{\sqrt{5}+1}{4}$$
$$\cos(\frac{9\pi}{10})=\cos(162°)=-\frac{\sqrt{10+2\sqrt{5}}}{4}$$

und

$$\sin(\frac{0\pi}{10})=\sin(0°)=0$$
$$\sin(\frac{1\pi}{10})=\sin(18°)=\frac{\sqrt{5}-1}{4}$$
$$\sin(\frac{2\pi}{10})=\sin(36°)=\frac{\sqrt{10-2\sqrt{5}}}{4}$$
$$\sin(\frac{3\pi}{10})=\sin(54°)=\frac{\sqrt{5}+1}{4}$$
$$\sin(\frac{4\pi}{10})=\sin(72°)=\frac{\sqrt{10+2\sqrt{5}}}{4}$$
$$\sin(\frac{5\pi}{10})=\sin(90°)=1$$
$$\sin(\frac{6\pi}{10})=\sin(108°)=\frac{\sqrt{10+2\sqrt{5}}}{4}$$
$$\sin(\frac{7\pi}{10})=\sin(126°)=\frac{\sqrt{5}+1}{4}$$
$$\sin(\frac{8\pi}{10})=\sin(144°)=\frac{\sqrt{10-2\sqrt{5}}}{4}$$
$$\sin(\frac{9\pi}{10})=\sin(162°)=\frac{\sqrt{5}-1}{4}$$

erhalten wir

$$[1x+0y]\left[\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y\right]\left[\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y\right]\left[\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y\right]$$
$$\left[\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y\right][0x+1y]\left[-\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y\right]\left[-\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y\right]$$
$$\left[-\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y\right]\left[-\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y\right]-z=0$$

$$x\,y\left[\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y\right]\left[\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y\right]\left[\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y\right]$$
$$\left[\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y\right]\left[-\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y\right]\left[-\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y\right]$$
$$\left[-\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y\right]\left[-\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y\right]-z=0$$

Jetzt wird das Ausmultiplizieren etwas komplizierter, deshalb gehen wir schrittweise vor. Wir führen A, B, C, D, E, F, G und H ein.

$$x\,y[A\cdot B\cdot C\cdot D\cdot E\cdot F\cdot G\cdot H]-z=0$$

$$A=\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y$$
$$B=\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y$$
$$C=\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y$$
$$D=\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y$$
$$E=-\frac{\sqrt{5}-1}{4}x+\frac{\sqrt{10+2\sqrt{5}}}{4}y$$
$$F=-\frac{\sqrt{10-2\sqrt{5}}}{4}x+\frac{\sqrt{5}+1}{4}y$$
$$G=-\frac{\sqrt{5}+1}{4}x+\frac{\sqrt{10-2\sqrt{5}}}{4}y$$
$$H=-\frac{\sqrt{10+2\sqrt{5}}}{4}x+\frac{\sqrt{5}-1}{4}y$$

Um das Ausmultiplizieren zu vereinfachen stellen wir die Gleichungen etwas um.

$$A=\frac{\sqrt{5}-1}{4}y+\frac{\sqrt{10+2\sqrt{5}}}{4}x$$
$$B=\frac{\sqrt{10-2\sqrt{5}}}{4}y+\frac{\sqrt{5}+1}{4}x$$
$$C=\frac{\sqrt{5}+1}{4}y+\frac{\sqrt{10-2\sqrt{5}}}{4}x$$
$$D=\frac{\sqrt{10+2\sqrt{5}}}{4}y+\frac{\sqrt{5}-1}{4}x$$
$$E=\frac{\sqrt{10+2\sqrt{5}}}{4}y-\frac{\sqrt{5}-1}{4}x$$
$$F=\frac{\sqrt{5}+1}{4}y-\frac{\sqrt{10-2\sqrt{5}}}{4}x$$
$$G=\frac{\sqrt{10-2\sqrt{5}}}{4}y-\frac{\sqrt{5}+1}{4}x$$
$$H=\frac{\sqrt{5}-1}{4}y-\frac{\sqrt{10+2\sqrt{5}}}{4}x$$

Jetzt nutzen wir die binomische Formel aus einer Formelsammlung.

$$(a+b)(a-b)=a^2-b^2$$

Danach müssen wir A mit H, B mit G, C mit F und D mit E multiplizieren.

$$I=A\cdot H=\left[\frac{\sqrt{5}-1}{4}y+\frac{\sqrt{10+2\sqrt{5}}}{4}x\right]\cdot\left[\frac{\sqrt{5}-1}{4}y-\frac{\sqrt{10+2\sqrt{5}}}{4}x\right]=\frac{(\sqrt{5}-1)^2}{16}y^2-\frac{10+2\sqrt{5}}{16}x^2$$

$$(\sqrt{5}-1)(\sqrt{5}-1)=5-2\sqrt{5}+1=6-2\sqrt{5}$$

$$I=A\cdot H=\frac{6-2\sqrt{5}}{16}y^2-\frac{10+2\sqrt{5}}{16}x^2$$

$$J=B\cdot G=\left[\frac{\sqrt{10-2\sqrt{5}}}{4}y+\frac{\sqrt{5}+1}{4}x\right]\cdot\left[\frac{\sqrt{10-2\sqrt{5}}}{4}y-\frac{\sqrt{5}+1}{4}x\right]=\frac{10-2\sqrt{5}}{16}y^2-\frac{(\sqrt{5}+1)^2}{16}x^2$$

$$(\sqrt{5}+1)(\sqrt{5}+1)=5+2\sqrt{5}+1=6+2\sqrt{5}$$

$$J=B\cdot G=\frac{10-2\sqrt{5}}{16}y^2-\frac{6+2\sqrt{5}}{16}x^2$$

$$K=C\cdot F=\left[\frac{\sqrt{5}+1}{4}y+\frac{\sqrt{10-2\sqrt{5}}}{4}x\right]\cdot\left[\frac{\sqrt{5}+1}{4}y-\frac{\sqrt{10-2\sqrt{5}}}{4}x\right]=\frac{(\sqrt{5}+1)^2}{16}y^2-\frac{10-2\sqrt{5}}{16}x^2$$

$$K=C\cdot F=\frac{6+2\sqrt{5}}{16}y^2-\frac{10-2\sqrt{5}}{16}x^2$$

$$L=D\cdot E=\left[\frac{\sqrt{10+2\sqrt{5}}}{4}y+\frac{\sqrt{5}-1}{4}x\right]\cdot\left[\frac{\sqrt{10+2\sqrt{5}}}{4}y-\frac{\sqrt{5}-1}{4}x\right]=\frac{10+2\sqrt{5}}{16}y^2-\frac{(\sqrt{5}-1)^2}{16}x^2$$

$$L=D\cdot E=\frac{10+2\sqrt{5}}{16}y^2-\frac{6-2\sqrt{5}}{16}x^2$$

Damit kommen wir dem Ergebnis einen Schritt näher.

$$x\,y[I\cdot J\cdot K\cdot L]-z=0$$

Im nächsten Schritt multiplizieren wir I mit J.

$$M=I\cdot J=\left[\frac{6-2\sqrt{5}}{16}y^2-\frac{10+2\sqrt{5}}{16}x^2\right]\cdot\left[\frac{10-2\sqrt{5}}{16}y^2-\frac{6+2\sqrt{5}}{16}x^2\right]$$

$$M=I\cdot J=\frac{(6-2\sqrt{5})(10-2\sqrt{5})}{256}y^4-\frac{(6-2\sqrt{5})(6+2\sqrt{5})}{256}x^2y^2$$

$$-\frac{(10+2\sqrt{5})(10-2\sqrt{5})}{256}x^2y^2+\frac{(10+2\sqrt{5})(6+2\sqrt{5})}{256}x^4$$

$$(6-2\sqrt{5})(10-2\sqrt{5})=60-12\sqrt{5}-20\sqrt{5}+20=80-32\sqrt{5}$$

$$(6-2\sqrt{5})(6+2\sqrt{5})=36-20=16$$

$$(10+2\sqrt{5})(10-2\sqrt{5})=100-20=80$$

$$(10+2\sqrt{5})(6+2\sqrt{5})=60+20\sqrt{5}+12\sqrt{5}+20=80+32\sqrt{5}$$

$$M=I\cdot J=\frac{80-32\sqrt{5}}{256}y^4-\frac{16}{256}x^2y^2-\frac{80}{256}x^2y^2+\frac{80+32\sqrt{5}}{256}x^4$$

$$M=I\cdot J=\frac{80-32\sqrt{5}}{256}y^4-\frac{96}{256}x^2y^2+\frac{80+32\sqrt{5}}{256}x^4$$

Im nächsten Schritt multiplizieren wir K mit L.

$$N=K\cdot L=\left[\frac{6+2\sqrt{5}}{16}y^2-\frac{10-2\sqrt{5}}{16}x^2\right]\cdot\left[\frac{10+2\sqrt{5}}{16}y^2-\frac{6-2\sqrt{5}}{16}x^2\right]$$

$$N=K\cdot L=\frac{(6+2\sqrt{5})(10+2\sqrt{5})}{256}y^4-\frac{16}{256}x^2y^2-\frac{80}{256}x^2y^2+\frac{(6-2\sqrt{5})(10-2\sqrt{5})}{256}x^4$$

$$N=K\cdot L=\frac{80+32\sqrt{5}}{256}y^4-\frac{96}{256}x^2y^2+\frac{80-32\sqrt{5}}{256}x^4$$

Damit kommen wir dem Ergebnis einen Schritt näher.

$$x\,y[M\cdot N]-z=0$$

Als letzten Schritt müssen wir M mit N multiplizieren.

$$M\cdot N=\left[\frac{80-32\sqrt{5}}{256}y^4-\frac{96}{256}x^2y^2+\frac{80+32\sqrt{5}}{256}x^4\right]\cdot\left[\frac{80+32\sqrt{5}}{256}y^4-\frac{96}{256}x^2y^2+\frac{80-32\sqrt{5}}{256}x^4\right]$$

$$(80-32\sqrt{5})(80+32\sqrt{5})=6400-5120=1280$$

$$(80-32\sqrt{5})(80-32\sqrt{5})=6400-5120\sqrt{5}+5120=11520-5120\sqrt{5}$$

$$(80+32\sqrt{5})(80+32\sqrt{5})=6400+5120\sqrt{5}+5120=11520+5120\sqrt{5}$$

$$M\cdot N=\frac{1280}{65536}y^8-\frac{7680-3072\sqrt{5}}{65536}x^2y^6+\frac{11520-5120\sqrt{5}}{65536}x^4y^4$$

$$-\frac{7680+3072\sqrt{5}}{65536}x^2y^6+\frac{9216}{65536}x^4y^4-\frac{7680-3072\sqrt{5}}{65536}x^6y^2$$

$$+\frac{11520+5120\sqrt{5}}{65536}x^4y^4-\frac{7680+3072\sqrt{5}}{65536}x^6y^2+\frac{1280}{65536}x^8$$

Vereinfachen der Gleichung.

$$-\frac{7680-3072\sqrt{5}}{65536}x^2y^6-\frac{7680+3072\sqrt{5}}{65536}x^2y^6=-\frac{15360}{65536}x^2y^6=-\frac{60}{256}x^2y^6$$

$$\frac{11520-5120\sqrt{5}}{65536}x^4y^4+\frac{9216}{65536}x^4y^4+\frac{11520+5120\sqrt{5}}{65536}x^4y^4=\frac{32256}{65536}x^4y^4=\frac{126}{256}x^4y^4$$

$$-\frac{7680-3072\sqrt{5}}{65536}x^6y^2-\frac{7680+3072\sqrt{5}}{65536}x^6y^2=-\frac{15360}{65536}x^6y^2=-\frac{60}{256}x^6y^2$$

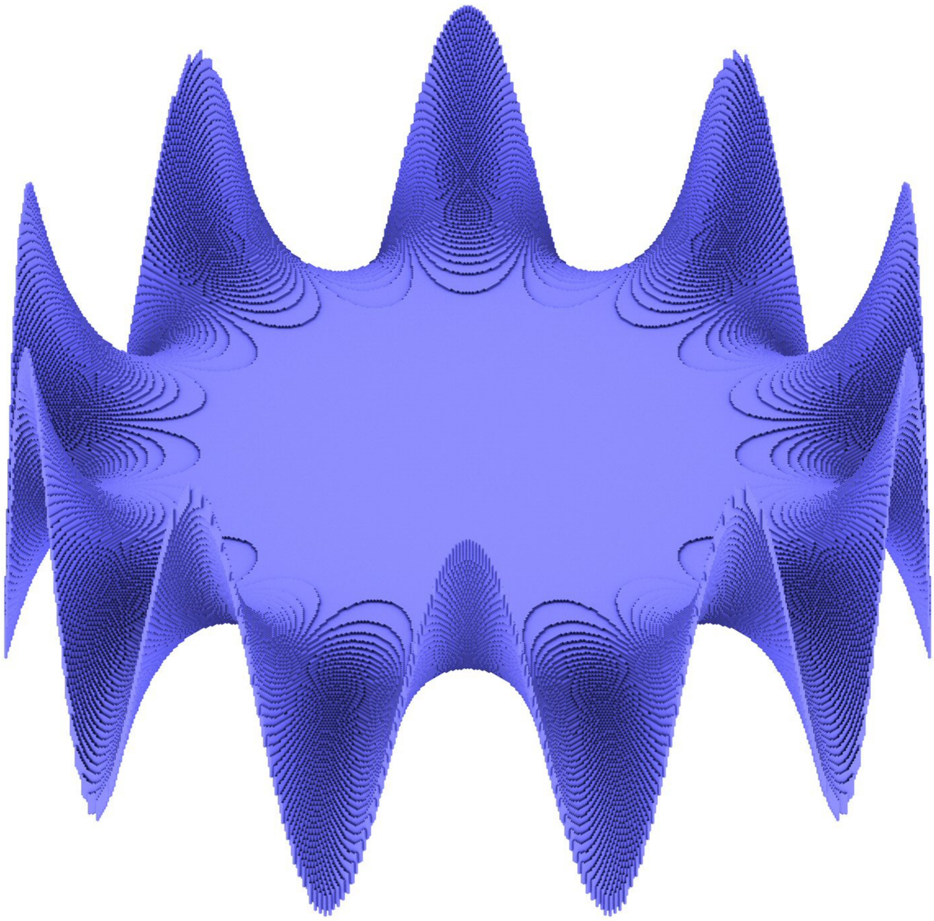
Für M mal N erhält man

$$M\cdot N=\frac{5}{256}y^8-\frac{60}{256}x^2y^6+\frac{126}{256}x^4y^4-\frac{60}{256}x^6y^2+\frac{5}{256}x^8$$

Damit erhält man für n = 10

$$x\,y\left[\frac{5}{256}x^8-\frac{60}{256}x^6y^2+\frac{126}{256}x^4y^4-\frac{60}{256}x^2y^6+\frac{5}{256}y^8\right]-z=0$$

Das Ergebnis der Formel.



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