$$\prod_{i=0}^{i<n} [x\cos(\frac{i\pi}{n}) + y\sin(\frac{i\pi}{n})] - z = 0$$

Für n = 4 gilt

$$[x\cos(\frac{0\pi}{4}) + y\sin(\frac{0\pi}{4})][x\cos(\frac{1\pi}{4}) + y\sin(\frac{1\pi}{4})][x\cos(\frac{2\pi}{4}) + y\sin(\frac{2\pi}{4})][x\cos(\frac{3\pi}{4}) + y\sin(\frac{3\pi}{4})] - z = 0$$

Mit

$$\cos(\frac{0\pi}{4}) = \cos(0°) = 1 \qquad \sin(\frac{0\pi}{4}) = \sin(0°) = 0$$
$$\cos(\frac{1\pi}{4}) = \cos(45°) = \frac{\sqrt{2}}{2} \qquad \sin(\frac{1\pi}{4}) = \sin(45°) = \frac{\sqrt{2}}{2}$$
und
$$\cos(\frac{2\pi}{4}) = \cos(90°) = 0 \qquad \sin(\frac{2\pi}{4}) = \sin(90°) = 1$$
$$\cos(\frac{3\pi}{4}) = \cos(135°) = -\frac{\sqrt{2}}{2} \qquad \sin(\frac{3\pi}{4}) = \sin(135°) = \frac{\sqrt{2}}{2}$$

erhalten wir

$$[1x + 0y]\left[\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right][0x + 1y]\left[-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right] - z = 0$$

$$x\,y\left[\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right]\left[-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right] - z = 0$$

Multiplizieren wir die beiden Klammern aus.

$$\left[\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right]\left[-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right] = -\frac{1}{2}x^2 + \frac{1}{2}x\,y - \frac{1}{2}x\,y + \frac{1}{2}y^2 = -\frac{1}{2}x^2 + \frac{1}{2}y^2$$

Damit erhält man für n = 4

$$x\,y\left[-\frac{1}{2}x^2 + \frac{1}{2}y^2\right] - z = 0$$

Das Ergebnis der Formel.