$$\prod_{i=0}^{i<n}[x\cos(\frac{i\pi}{n})+y\sin(\frac{i\pi}{n})]-z=0$$

Für n = 8 gilt

$$[x\cos(\frac{0\pi}{8})+y\sin(\frac{0\pi}{8})][x\cos(\frac{1\pi}{8})+y\sin(\frac{1\pi}{8})]$$
$$[x\cos(\frac{2\pi}{8})+y\sin(\frac{2\pi}{8})][x\cos(\frac{3\pi}{8})+y\sin(\frac{3\pi}{8})]$$
$$[x\cos(\frac{4\pi}{8})+y\sin(\frac{4\pi}{8})][x\cos(\frac{5\pi}{8})+y\sin(\frac{5\pi}{8})]$$
$$[x\cos(\frac{6\pi}{8})+y\sin(\frac{6\pi}{8})][x\cos(\frac{7\pi}{8})+y\sin(\frac{7\pi}{8})]-z=0$$

Mit

$$\cos(\frac{0\pi}{8})=\cos(0°)=1$$
$$\cos(\frac{1\pi}{8})=\cos(22{,}5°)=\frac{\sqrt{2+\sqrt{2}}}{2}$$
$$\cos(\frac{2\pi}{8})=\cos(45°)=\frac{\sqrt{2}}{2}$$
$$\cos(\frac{3\pi}{8})=\cos(67{,}5°)=\frac{\sqrt{2-\sqrt{2}}}{2}$$
$$\cos(\frac{4\pi}{8})=\cos(90°)=0$$
$$\cos(\frac{5\pi}{8})=\cos(112{,}5°)=-\frac{\sqrt{2-\sqrt{2}}}{2}$$
$$\cos(\frac{6\pi}{8})=\cos(135°)=-\frac{\sqrt{2}}{2}$$
$$\cos(\frac{7\pi}{6})=\cos(157{,}5°)=-\frac{\sqrt{2+\sqrt{2}}}{2}$$

und

$$\sin(\frac{0\pi}{8})=\sin(0°)=0$$
$$\sin(\frac{1\pi}{8})=\sin(22{,}5°)=\frac{\sqrt{2-\sqrt{2}}}{2}$$
$$\sin(\frac{2\pi}{8})=\sin(45°)=\frac{\sqrt{2}}{2}$$
$$\sin(\frac{3\pi}{8})=\sin(67{,}5°)=\frac{\sqrt{2+\sqrt{2}}}{2}$$
$$\sin(\frac{4\pi}{8})=\sin(90°)=1$$
$$\sin(\frac{5\pi}{8})=\sin(112{,}5°)=\frac{\sqrt{2+\sqrt{2}}}{2}$$
$$\sin(\frac{6\pi}{8})=\sin(135°)=\frac{\sqrt{2}}{2}$$
$$\sin(\frac{7\pi}{8})=\sin(157{,}5°)=\frac{\sqrt{2-\sqrt{2}}}{2}$$

erhalten wir

$$[1\mathrm{x}+0\mathrm{y}]\left[\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y\right]\left[\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y\right]\left[\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y\right]$$
$$[0\mathrm{x}+1\mathrm{y}]\left[-\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y\right]\left[-\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y\right]\left[-\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y\right]-z=0$$

$$xy\left[\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y\right]\left[\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y\right]\left[\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y\right]$$
$$\left[-\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y\right]\left[-\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y\right]\left[-\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y\right]-z=0$$

Jetzt wird das Ausmultiplizieren etwas komplizierter, deshalb gehen wir schrittweise vor. Wir führen A, B, C, D, E und F ein.

$$x\,y[A\cdot B\cdot C\cdot D\cdot E\cdot F]-z=0$$

mit

$$A=\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y$$
$$B=\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y$$
$$C=\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y$$
$$D=-\frac{\sqrt{2-\sqrt{2}}}{2}x+\frac{\sqrt{2+\sqrt{2}}}{2}y$$
$$E=-\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y$$
$$F=-\frac{\sqrt{2+\sqrt{2}}}{2}x+\frac{\sqrt{2-\sqrt{2}}}{2}y$$

Um das Ausmultiplizieren zu vereinfachen stellen wir die Gleichungen etwas um.

$$A=\frac{\sqrt{2-\sqrt{2}}}{2}y+\frac{\sqrt{2+\sqrt{2}}}{2}x$$
$$B=\frac{\sqrt{2}}{2}y+\frac{\sqrt{2}}{2}x$$
$$C=\frac{\sqrt{2+\sqrt{2}}}{2}y+\frac{\sqrt{2-\sqrt{2}}}{2}x$$
$$D=\frac{\sqrt{2+\sqrt{2}}}{2}y-\frac{\sqrt{2-\sqrt{2}}}{2}x$$
$$E=\frac{\sqrt{2}}{2}y-\frac{\sqrt{2}}{2}x$$
$$F=\frac{\sqrt{2-\sqrt{2}}}{2}y-\frac{\sqrt{2+\sqrt{2}}}{2}x$$

Jetzt nutzen wir die binomische Formel aus einer Formelsammlung.

$$(a+b)(a-b)=a^2-b^2$$

Danach müssen wir A mit F, B mit E und C mit D multiplizieren.

$$G=A\cdot F=\left[\frac{\sqrt{2-\sqrt{2}}}{2}y+\frac{\sqrt{2+\sqrt{2}}}{2}x\right]\cdot\left[\frac{\sqrt{2-\sqrt{2}}}{2}y-\frac{\sqrt{2+\sqrt{2}}}{2}x\right]=\frac{2-\sqrt{2}}{4}y^2-\frac{2+\sqrt{2}}{4}x^2$$

$$H=B\cdot E=\left[\frac{\sqrt{2}}{2}y+\frac{\sqrt{2}}{2}x\right]\cdot\left[\frac{\sqrt{2}}{2}y-\frac{\sqrt{2}}{2}x\right]=\frac{1}{2}y^2-\frac{1}{2}x^2$$

$$I=C\cdot D=\left[\frac{\sqrt{2+\sqrt{2}}}{2}y+\frac{\sqrt{2-\sqrt{2}}}{2}x\right]\cdot\left[\frac{\sqrt{2+\sqrt{2}}}{2}y-\frac{\sqrt{2-\sqrt{2}}}{2}x\right]=\frac{2+\sqrt{2}}{4}y^2-\frac{2-\sqrt{2}}{4}x^2$$

Damit kommen wir dem Ergebnis einen Schritt näher.

$$x\,y[G\cdot H\cdot I]-z=0$$

Im nächsten Schritt multiplizieren wir G mit I.

$$J=G\cdot I=\left[\frac{2-\sqrt{2}}{4}y^2-\frac{2+\sqrt{2}}{4}x^2\right]\cdot\left[\frac{2+\sqrt{2}}{4}y^2-\frac{2-\sqrt{2}}{4}x^2\right]$$

$$J=\frac{(2-\sqrt{2})(2+\sqrt{2})}{16}y^4-\frac{(2-\sqrt{2})(2-\sqrt{2})}{16}y^2x^2-\frac{(2+\sqrt{2})(2+\sqrt{2})}{16}x^2y^2+\frac{(2+\sqrt{2})(2-\sqrt{2})}{16}x^4$$

$$(2-\sqrt{2})(2+\sqrt{2})=4-2=2$$

$$(2-\sqrt{2})(2-\sqrt{2})=4-2\sqrt{2}-2\sqrt{2}+2=6-4\sqrt{2}$$

$$(2+\sqrt{2})(2+\sqrt{2})=4+2\sqrt{2}+2\sqrt{2}+2=6+4\sqrt{2}$$

$$J=\frac{2}{16}y^4-\frac{6-4\sqrt{2}}{16}y^2x^2-\frac{6+4\sqrt{2}}{16}x^2y^2+\frac{2}{16}x^4$$

$$J=\frac{2}{16}y^4+\frac{-6+4\sqrt{2}}{16}y^2x^2+\frac{-6-4\sqrt{2}}{16}x^2y^2+\frac{2}{16}x^4$$

$$J=\frac{2}{16}y^4+\frac{-12}{16}x^2y^2+\frac{2}{16}x^4$$

$$J=\frac{1}{8}y^4-\frac{3}{4}x^2y^2+\frac{1}{8}x^4$$

Als letzten Schritt müssen wir H mit J multiplizieren.

$$H\cdot J=\left[\frac{1}{2}y^2-\frac{1}{2}x^2\right]\cdot\left[\frac{1}{8}y^4-\frac{3}{4}x^2y^2+\frac{1}{8}x^4\right]$$

$$H\cdot J=\frac{1}{16}y^6-\frac{3}{8}x^2y^4+\frac{1}{16}x^4y^2-\frac{1}{16}x^2y^4+\frac{3}{8}x^4y^2-\frac{1}{16}x^6$$

$$H\cdot J=\frac{1}{16}y^6-\frac{6}{16}x^2y^4+\frac{1}{16}x^4y^2-\frac{1}{16}x^2y^4+\frac{6}{16}x^4y^2-\frac{1}{16}x^6$$

$$H\cdot J=\frac{1}{16}y^6-\frac{7}{16}x^2y^4+\frac{7}{16}x^4y^2-\frac{1}{16}x^6$$

Damit erhält man für n = 8

$$x\,y\left[-\frac{1}{16}x^6+\frac{7}{16}x^4y^2-\frac{7}{16}x^2y^4+\frac{1}{16}y^6\right]-z=0$$

Das Ergebnis der Formel.